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MASLOV'S INVERSE METHOD AND DECIDABLE CLASSES

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The inverse method proposed by S.Yu. Maslov [5] simultaneously with the resolution principle [3] but independently of it, has turned out to be a very powerful tool of proof search theory. On the basis of the inverse method for the first time there has been developed and described [6] a scheme which uniformises proofs of decidability for many wellknown classes of formulas, their generalizations and also for some new decidable classes.

This scheme was used later in the resolution method too [4], [8].

The principal idea of the decidability proof with this scheme is the finiteness of the search space for the proof search by the inverse method for every formula of the decidable class.

The aim of this paper is to give a complete and self-contained proof of the decidability for the class K [6]. This class contains several known decidable classes (Ackermann, Gödel–Kalmar–Schütte classes etc.). The proof presented here is based on the inverse method only. All necessary definitions are formulated and all necessary results are proved (including ones concerning the inverse method). The readers familiar with the resolution method should remember that we work in terms of deducibility (provability) while the resolution principle is formulated in terms of satisfiability.

We will consider the Skolelm normal form of prenex formulas that is the result of eliminating universal quantifiers: every subformula $\exists x_1 \cdots \exists x_k \forall y A(x_1, \ldots, x_k, y)$ is replaced by the formula $\exists x_1 \cdots \exists x_k A(x_1, \ldots, x_k, f(x_1, \ldots, x_k))$ where f is a new functional symbol.

In this way we obtain a formula containing existential quantifiers only.

We will consider formulas in prenex normal form with a matrix in disjunctive normal form.

By $\exists(F)$ we denote the formula $\exists x_1 \cdots \exists x_k F$, where $x_1, \ldots, x_k$ are all variables of F. Analogous by $\forall(F)$ we denote the formula $\forall x_1 \cdots \forall x_k F$.

In Section 1 we describe a version of the inverse method and some search strategies. In Section 2 we introduce the class K of formulas in terms of restrictions on quantifier prefixes and a class KS which is a result of reduction of